

Cost-Revenue-Profit Functions (Using Linear Equations)

Profit maximization and Cost minimization are fundamental concepts in Business and Economic Theory. This handout is formatted to explain the process of understanding, creating, and interpreting cost-revenue-profit functions. When paired with the handout “**Marginal Analysis of Cost-Revenue-Profit Functions**” and the concepts learned in Business Calculus, this handout will provide a foundation of the Business and Economic Theory needed to understand how to maximize profits and minimize costs by using cost-revenue-profit functions.

Note: Cost, revenue, and profit functions don't only have to be in linear form. For ease of defining the functions we will work with equations in linear form because it is a bit more straightforward, but we can't neglect working these types of problems with quadratic equations [ie $F(x) = ax^2 + bx + c$]. For more information on quadratic cost-revenue-profit functions see the handout “**Marginal Analysis of Cost-Revenue-Profit Functions**”.

Cost functions

Cost functions tell us what the total cost of producing output is. The cost function consists of two different types of cost: Variable costs and fixed costs.

Variable cost varies with output (the number of units produced).

Ex: It costs a t-shirt making company \$5 to make each shirt.

Fixed cost will be the same no matter what the output is.

Ex: It costs the same t-shirt company \$250 a week to operate all the equipment (machines, lights, electricity, etc.) to make t-shirts, regardless of how many t-shirts are made that week.

Output is defined as what/how-much is being produced and is usually represented either as the variable “ x ” or as “ q ” (for quantity). But it can be represented with any letter, and regardless of the letter, all problems will be solved the same way.

(ex: an auto manufacturer's output is number of cars, a dry cleaner's output is number of clean clothes)

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A cost function represents the **total cost** (*fixed cost + variable cost*), and is often expressed as a linear equation [i.e. = $mx + b$].

$$\text{Total Cost, } C(x) \rightarrow C(x) = (m * x) + b = (\text{Variable Cost} * \text{Output}) + \text{Fixed Cost}$$

Fixed cost is assigned to b

Variable cost is assigned to m

Output is assigned to x

Let's analyze a cost function by working through a typical word problem that we would come across

Example:

A firm that makes pencils plans on **producing 500 pencils a week**. Based off their production costs throughout the years, they figure that it will cost them **\$1.00 to make each pencil**. There will also be a cost for running the equipment for a week that they figure to be **\$350 for the whole week, regardless of how many pencils they make**.

What is their fixed cost (b)?

\$350

What is their variable cost (m)?

\$1.00

What is their output (x)?

500

So with all this in mind, our total cost can be written in the cost equation form:

$$C(x) = 1.00x + 350$$

Let's plug in the output into our cost function so we can see what **the total cost of producing 500 pencils is**:

$$\begin{aligned} C(500) &= 1.00(500) + 350 \\ &= \$850 \end{aligned}$$

It costs \$850 to produce 500 pencils in one week

Also notice that producing one extra unit of output ($x = 501$) would raise our cost by \$1 (remember that \$1 is our variable cost) from \$850 to \$851. This is the **marginal cost** of this particular cost function. For each additional unit of x sold, our cost increases by \$1 (again: selling 500 pencils costs \$850, selling 501 pencils costs \$851)

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Revenue Functions

Revenue is the total payment received from selling a good, performing a service, etc.

Warning: Don't confuse revenue with profit though, we will define profit very soon and will see why they aren't the same thing.

The revenue function, $R(x)$, reflects the revenue from selling “ x ” amount of output items at a price of “ p ” per item.

$$R(x) = px$$

Example:

The same pencil factory decides to sell its 500 pencils for \$1.75 each. What would the total revenue be?

What is the price (p)? **\$1.75**

What is the output (x)? **500**

$$R(500) = (\$1.75) * (500) = \$875$$

Notice that producing one extra unit of output ($x = 501$) would raise our revenue by our selling price of \$1.75 (selling 500 pencils gains us a revenue of \$875, and selling 501 pencils gains us a revenue of \$876.75). This is the **marginal revenue** for this particular revenue function, $R(x)$. For each additional unit of x sold, our revenue increases by the selling price (in this case \$1.75).

Profit Functions

In business and economic courses we learn that profit and revenue are not the same thing. Revenue reflects how much we earn from selling our product (gross proceeds) without reflecting what the costs were. Profit functions, however, takes into account the costs of production and represents **how much the company really makes after deducting the costs of production from the revenue**. This is why Profit is defined as:

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$P = R - C \quad \text{or} \quad P(x) = R(x) - C(x)$$

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Example:

We combine the revenue and cost functions that we found for the pencil company to realize the Profit function for this company and figure out how much profit they obtain from making and selling 500 pencils.

Remember that: $R(x) = 1.75x$ and $C(x) = 1.00x + 350$

We can either:

1). **Plug in the formulas, combine like terms, and then solve for when $x = 500$**

$$P(x) = R(x) - C(x)$$

$$P(x) = (1.75x) - (1.00x + 350)$$

$$P(x) = .75x - 350$$

$$P(500) = (.75 * 500) - 350 = 375 - 350 = \mathbf{\$25}$$

2). **Or, solve the equations independently and then combine them to solve for $P(500)$**

$$C(500) = \$850 \qquad R(500) = \$875$$

$$P(500) = R(500) - C(500)$$

$$P(500) = 875 - 850 = \mathbf{\$25}$$

The company gains a profit of \$25 from selling 500 pencils. If we sold one more pencil ($x=501$), then our profit would be \$25.75. Our **marginal profit** would be \$.75 (\$25.75 - \$25). Therefore, each additional unit of x sold, our profit increases by \$.75 (increasing x by one from 500 to 501, increases our profit by our marginal profit of \$.75, from \$25 to \$25.75). Notice that \$.75 is also the slope of the Profit function, $P(x) = .75x - 350$.

Alternatively, the **marginal profit** is also **the marginal revenue** (\$.75 in our example) **minus our marginal cost** (\$1.00 in our example). Our profits will increase by \$.75 each time we add another unit of x to be sold.

(i.e. Marginal Profit = Marginal Revenue – Marginal Cost)

Another important point of the Profit function is the **break-even point**: where we neither make a profit nor a loss [$P(x) = 0$]. This occurs where $R = C$

So using our equations to fill out: $R(x) = C(x) \rightarrow 1.75x = 1.00x + 350$

We can determine the amount of output that would lead to the company's profits breaking even by solving for x .

$$1.75x = 1.00x + 350 \rightarrow .75x = 350 \rightarrow x = 466.66$$

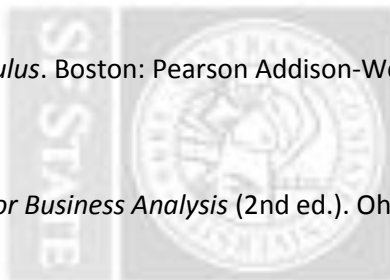
We will break even (have a profit of zero) if we produce approximately 467 pencils (round up from 466.66).

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References

Hass, J., & Weir, M. (2007). *University Calculus*. Boston: Pearson Addison-Wesley.

Waner, S., & Costenoble, S. (2009). *Math for Business Analysis* (2nd ed.). Ohio: Cengage Learning.



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