

# Campus Academic Resource Program

## Introduction to Fractions

### Fractions Help: What you need to know.

Welcome to the *Fractions Help* handout! This is a tool for you to use to remember the basics and properties of fractions and how to approach problems dealing with fractions.

**(In order to get the most out of this handout, you should be familiar with what *Factors* and *Multiples* of numbers are.)**

This handout will:

1. Define Key Terms and Properties of Fractions
2. Cover Adding and Subtracting Fractions
  - Common Denominators
3. Cover Multiplying Fractions
  - Cross Cancelling
4. Cover Dividing Fractions
  - Reciprocals
5. Provide an Appendix I: Lowest Common Denominator
6. Provide Outside Resources
  - Simplifying, Lowest Common Denominator, Greatest Common Factor

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### Important terms

**Numerator**- What we call the number found on the *top* of the fraction.

$$\frac{2}{3} \quad \begin{array}{l} 2 \text{ is the } \mathbf{numerator} \\ 3 \text{ is the } \mathbf{denominator} \end{array}$$

**Denominator**- What we call the number on the *bottom* of the fraction.

**Proper Fraction**- This refers to a fraction where the *smaller* number is the numerator. *Example:*  $\frac{2}{3}$

**Improper fraction**- This refers to a fraction where the *larger* number is the numerator.

We prefer to leave fractions improper, do not use a mixed number<sup>1</sup> as an answer. *Example:*  $\frac{3}{2}$

**Common Denominator (CD)**<sup>2</sup> - This is any multiple of two *non-like denominators*, within an equation. A multiple diagram<sup>3</sup> is commonly made to represent the multiples of each denominator.

*We use this property in **addition/subtraction of fractions***

*See link at the end for more info.*

3	3, 6, 9, <b>12</b> , 15, 18, 21, <b>24</b>
4	4, 8, <b>12</b> , 16, <b>24</b> , 28

*example:*  $\frac{1}{4}$  &  $\frac{1}{3}$  have the common denominators: **12 and 24**

**Like Fractions**- This is when two or more fractions have the same *denominators*.

*We use this property in the **addition/subtraction of fractions**.*

*examples:*  $\frac{1}{4}$  &  $\frac{3}{4}$  &  $\frac{5}{4}$

**Fancy One**- This is a term we will use (in this handout only) to identify the fraction representation of 1.

*We use this property in finding **common denominators**.*

*example:*  $\frac{a}{a} = 1$  &  $\frac{2}{2} = 1$

**Reciprocal**- This is a fraction with its numerator and denominator flipped from the original.

*We use this property in the **division of fractions**.*

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<sup>1</sup> **Mixed number**- when an improper fraction is represented as a *whole number with a fraction*. *example:*  $\frac{3}{2} = 1\frac{1}{2}$

<sup>2</sup> It is common to pick the smallest of the common denominators—known as the **LCD**. See Appendix I.

<sup>3</sup> Multiple diagram is defined on page 3 under “Factor/Multiple Diagram”

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*Example:*  $\frac{2}{3}$  has the reciprocal  $\frac{3}{2}$

**Lowest Common Denominator<sup>4</sup> (LCD)**- This is the smallest multiple shared by two denominators. It is helpful in combining non-like fractions in the addition and subtraction of fractions.

*We can use this in the **addition/subtraction of fractions**.*

The LCD can be found using the *Multiple diagram*.

**Greatest Common Factor (GCF)**- This is the largest number which can divide both the numerator and denominator.

*We use this to carry out **simplification of fractions**.*

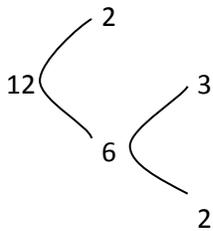
The GCF can be found using two methods: *Factor diagram* and *Prime Factorization*.

**Factor/Multiple Diagram**- This is what the handout will call the boxes used to organize the factors and multiples of our denominators. The matching values are bolded to emphasize the answers.

Number	Factors/multiples <b>matches</b>
Number	Factors/multiples <b>matches</b>

**Prime Factorization**- This is the use of a “factor tree” to help find the factors of a number. It uses a branching method of dividing a number by small factors, and dividing those factors if possible, until all prime numbers remain.

*We use this method to find the **GCF** that is then useful in **simplifying fractions**.*



The factors of 12 are 2, 3. (The 2 is repeated, so we just say it once.)

**Simplifying**- This is the practice of representing a fraction with the simplest values. In order to carry this out, you must find and eliminate the **common factors** between the numerator and denominator. This leaves a reduced representation of the same value.

*We use this generally at the end of each problem dealing with fractions.*

*See the link in outside resources for more info.*

*Example:*  $\frac{2}{4}$  has a common factor of 2. Simplifying to  $\frac{(2\div 2)}{(4\div 2)} = \frac{1}{2}$

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<sup>4</sup> See Appendix I: Lowest Common Denominator.

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### The Properties of Adding/Subtracting Fractions

1.) Only **like fractions** can be combined: this means fractions must have **common denominators** to be added/subtracted.

a. If the fractions aren't matching, you must find the **CD**: here is a table to organize the multiples of 2 and 3

$$\text{Example: } \frac{1}{2} + \frac{2}{3} = \frac{3 \cdot 1}{3 \cdot 2} + \frac{2 \cdot 2}{2 \cdot 3} = \frac{3}{6} + \frac{4}{6}$$

Multiples of 2 & 3	
2	2, 4, 6, 8
3	3, 6, 9, 12

(We must use the **fancy one** to achieve the **CD** by multiplying the numerators and denominators by the same number.)

2.) Once denominators match, add/subtract across the *top only*.

$$\text{Example: } \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$$

(Notice the fraction is an **improper fraction**. That's okay because it's the most commonly accepted answer.)

3.) If the numerator and denominator share a common factor, find the *GCF* and **simplify**.

*Example:*

$$\frac{1}{4} + \frac{2}{5} = \left( \frac{5 \cdot 1}{5 \cdot 4} \right) + \left( \frac{2 \cdot 4}{5 \cdot 4} \right) = \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$$

(Here we apply the **fancy one**.)

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### Properties of Multiplying Fractions

1.) Here is the first of two methods commonly used to multiply fractions:

- a. Multiply both the *top and bottom* across both fractions.

$$\text{Example: } \frac{1}{2} \cdot \frac{4}{5} = \frac{1 \cdot 4}{2 \cdot 5} = \frac{4}{10}$$

- b. Factor out the ***greatest common factor*** from both numerator and denominator to simplify.

$$\text{Example: } \frac{1}{2} \cdot \frac{4}{5} = \frac{4}{10} = \frac{2}{5} \cdot \frac{2}{2} = \frac{2}{5}$$

GCF is: 2	
4	1, 2, 4
10	1, 2, 5, 10

2.) The alternate way is simply to avoid ***simplifying*** at the end:

- a. *Cross cancel* terms with common factors before you multiply.

- b. Then multiplying across the top and bottom, the resulting fraction should be simplified.

$$\text{Example: } \frac{1}{2} \cdot \frac{4}{5} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}}{5} = \frac{1}{1} \cdot \frac{2}{5} = \frac{2}{5}$$

### Properties of Dividing Fractions

The most important property used in division of fractions is the ***reciprocal***, which is defined in the terms.

The reciprocal of a fraction is just flipping the numerator and the denominator:

$$\frac{a}{b} \text{ has a reciprocal of } \frac{b}{a}$$

Keep in mind that whole numbers have an invisible 1 as a denominator; because any number divided by 1 is itself.

1.) Find the ***reciprocal*** of the fraction in the denominator.

$$\text{Example: } \frac{1/3}{2/3} \text{ reciprocal of } \frac{2}{3} \text{ is: } \frac{3}{2}$$

2.) Take the fraction in the *numerator* and multiply it by the ***reciprocal*** fraction of the *denominator*.

(This step is commonly referred to as “flip and multiply”).

$$\text{Example: } \frac{1/3}{2/3} = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2} = \frac{1}{2} \text{ here we cross cancelled the like values before multiplying.}$$

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### Appendix I: How to Find and Apply the LCD

- The **Lowest Common Denominator** is the smallest multiple that both of the denominators share.

We can find the **Lowest Common Denominator** within the *common denominators* we find when we are asked to add or subtract fractions with unlike denominators.

**\*\*In order to combine fractions by addition and subtraction, the denominators must match\*\***

- Take the two denominators and write out their multiples in a **Multiple Diagram**, find the *smallest matching number*: **LCD**

2	2, 4, <b>6</b> , 8, 12, 14, 16
3	3, <b>6</b> , 9, 12, 15, 18

- Once the LCD has been found, take the multiple and figure out what to multiply each denominator by to achieve it.

*Example:*  $\frac{1}{2} + \frac{1}{3}$  **LCD = 6** so  $\frac{3}{3} \cdot \frac{1}{2} = \frac{3}{6}$  &  $\frac{2}{2} \cdot \frac{1}{3} = \frac{2}{6}$  *Therefore:*  $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

- In order to create **like fractions** we must use our **fancy one** to modify each numerator and denominator.
- We can now combine the top of the fractions while keeping our LCD consistent throughout the bottom.

Practice with Finding the LCD:

a.) LCD of  $\frac{1}{2} + \frac{1}{4} =$


b.) LCD of  $\frac{3}{4} + \frac{2}{3} =$


c.) LCD of  $\frac{2}{5} - \frac{1}{5} =$


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### Outside Resources

Simplifying: <http://www.mathsisfun.com/simplifying-fractions.html>

Common denominators: <http://www.mathmistakes.info/facts/AlgebraFacts/learn/fr/add.html>

Greatest Common Factor: <http://www.mathsisfun.com/greatest-common-factor.html>

These links are here for more information on the theory of these practices and why they are helpful in dealing with fractions. These should also provide you a more in depth explanation of how to find the common denominators, greatest common factors, and how to simplify fractions.

Please make an appointment with CARP if you have any questions about this handout.

**Answers to Appendix Practice:** a.) 4      b.) 12      c.) 5

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### Citation

n/a. (2013, June 20). Math SOL 4.5 Add/Subtract Fractions 2013. . Retrieved July 21, 2014, from <http://www.lcps.k12.va.us/education/components/scrapbook/default.php?sectiondetailid=24271>

Pierce, R. (2013, January 1). Prime Factorization. Retrieved August 14, 2014, from <http://www.mathsisfun.com/prime-factorization.html>

Weisstein, Eric W. "Fraction." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com/Fraction.html>