This handout will:

- Define Linear and Quadratic Functions both graphically and algebraically
- Examine the associated equations and their components.
- Look at how each component could affect shape graphically
- Observe a Business Application of these functions

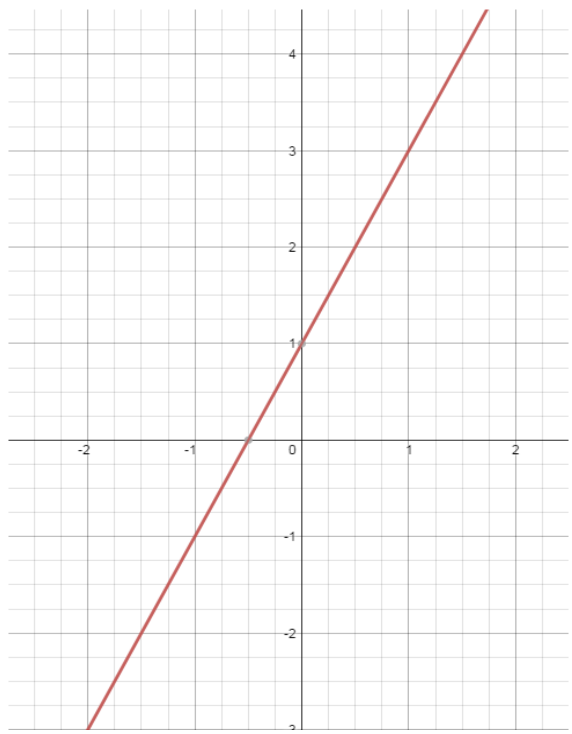
Linear Functions –

A linear function is a common function that represents *a straight line*

Algebraically: a linear function is associated with an equation which can be written in the form of:

$$F(x) = y = mx + b$$

Graphically: a linear function appears as a line.



Example:

$$F(x) = y = 2x + 1$$

Notice: m=2, b=1

The components:

Slope (m) – the slope of a line tells us how *"steep"* the line is. The steepness dictates how much "y" will increase or decrease when we change the value of x. Let's look at the example above ($y = 2x + 1$)

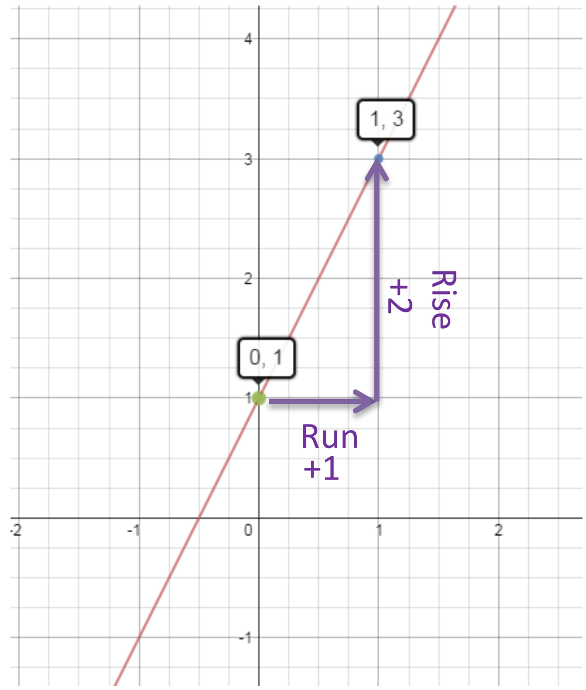
How much does "y" change by when "x" goes from 0 to 1?

If we pick *any two points* on the line, we can figure out the steepness of the line.

When "x" goes from 0 to 1, "y" goes from 1 to 3.

So, each time we increase x by 1, y increases by 2.

This means that our slope is 2 → **slope=m=2**



Could we have just used the equation to know the slope? Yes, because we saw from our formula that $m = 2$. We did a little bit of extra work, but that is ok and probably better, because sometimes we will not be given the equation.

Sometimes only two points will be provided. How would we find the slope then?

The **equation for finding slope** is:

$$\textbf{slope} = \mathrm{m} = \frac{\mathrm{y_2 - y_1}}{\mathrm{x_2 - x_1}}$$, given two sets of points (x_1, y_1) and (x_2, y_2)

It is also often shown as:

$$\mathrm{m} = \frac{\text{Rise}}{\text{Run}} \quad \text{or} \quad \mathrm{m} = \frac{\Delta \mathrm{y}}{\Delta \mathrm{x}} \quad \text{or} \quad \mathrm{m} = \frac{\text{Change in y}}{\text{Change in X}}$$

Though they are written differently, they all mean the same thing: in order to figure out the slope of a line, you must use two different points and *evaluate the changes in y over the changes in x.*

Let's use the points from the line above, (0,1) and (1.3), to solve for slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{1-0} = \frac{2}{1} = 2$$

We got the same answer!

Variable (x) – This is the **input** of the function. This means that any value we plug in is going to be substituted in for **x** (**x** is just a place holder that we replace with different numbers). The **output** will be the **y** value we get at the end and both the value for the **input** (**x**) and the value for the **output** (**y**) will **correspond to a particular point** on the line of the equation we are dealing with.

***Example*:**

Let's say our input is: x=2

For the function: $F(x) = 4x - 3$

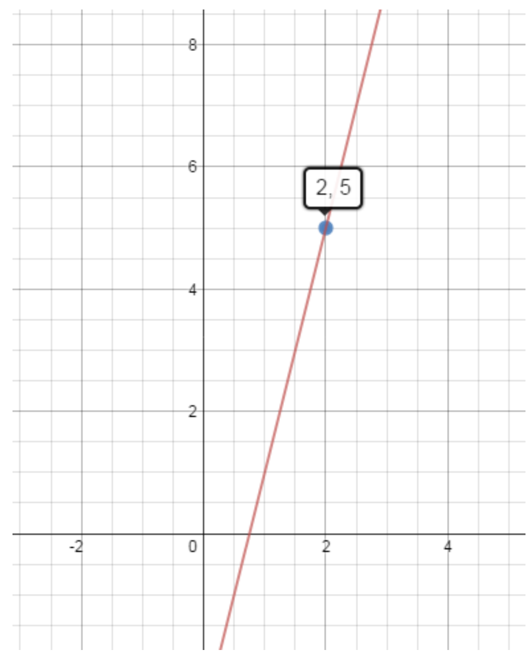
$$F(2) = 4(2) - 3$$

$$F(2) = 8 - 3 = 5$$

This corresponds to the point on the line: **(2,5)**

Because **x=2**, and when we plugged it in

F(2) = **y = 5**



Y- Intercept (b) – is Y's value when x = 0. Visually this is where the line will cross over the y-axis.

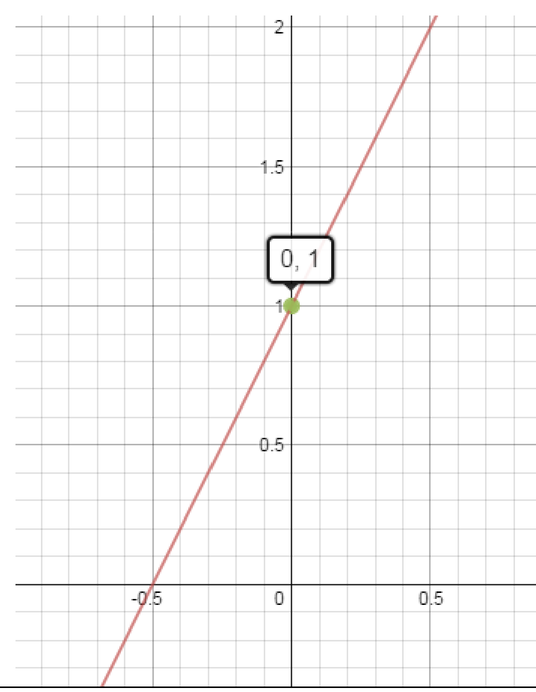
If x=0 for our general form equation y=mx+b then:

$y = (m \times 0) + b \rightarrow y = 0 + b \rightarrow \boldsymbol{y = b}$

In the equation, $y = 2x + 1$, we observed that b = 1, correct? Well, looking at the graph, what does y equal when the line is crossing the y axis?

It equals 1 doesn't it?

Note: you may have also heard the y-intercept be called the "Initial Value**".*



Quadratic Functions –

A quadratic function is a function that represents a *parabola (U-shape)*.

Algebraically: a quadratic function appears as an equation which can be written in the form:

$$f(x) = ax^2 + bx + c$$

where a, b, and c are fixed numbers and a≠0.

Example:

$$f(x) = 3x^2 - 4x - 1$$
$$(a = 3, b = -4, and\ c = -1)$$

Graphically: a quadratic function appears as a U-shape called a parabola.

Example:

$$y = x^2$$

(This is the most elementary form of a parabola.)

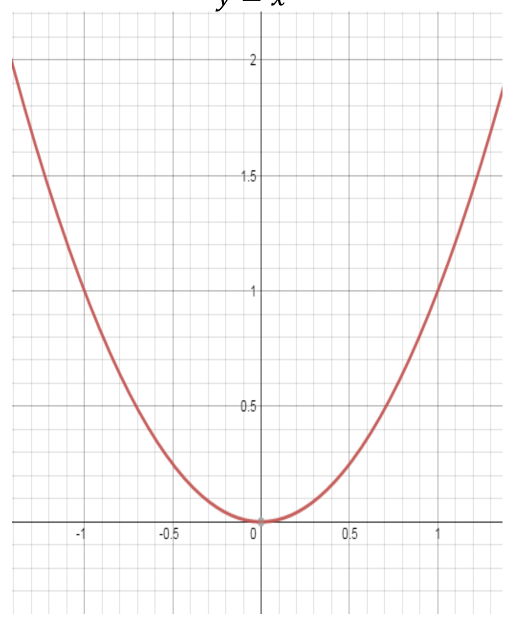
Note that for $y = x^2$:

a = 1, b = 0, c = 0

$$y = (1)x^2 + (0)x + (0) = x^2$$

Ex:

$y = x^2$

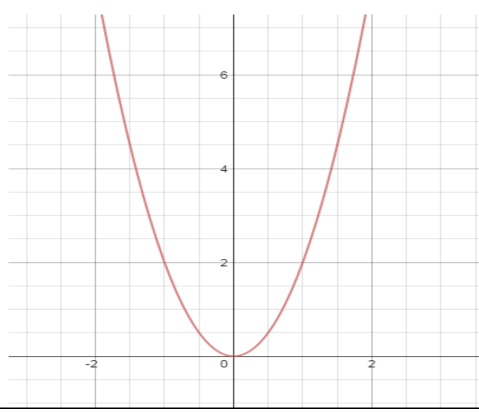


The Components:

"**a**": determines if the graph opens *upward or downward*, as well as how *thin or wide* the parabola is:

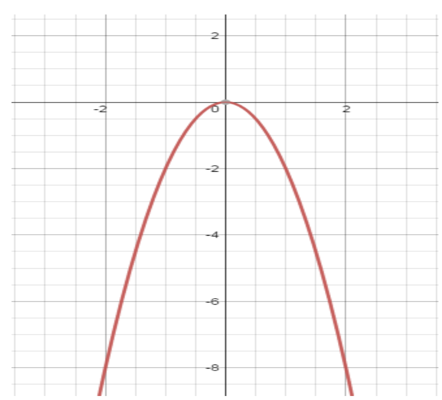
If **a** is *positive*, the parabola will open upwards as such:

$y = 2x^2$



If **a** is *negative*, the parabola will open downwards as such:

$y = -2x^2$



"**a**" is a bit similar to the slope in the sense that it changes the direction of the graph.

Also a *larger* absolute value of "**a**", where $a < -1$ *or* $a > 1$, makes the parabola *thinner* and a *smaller* absolute value of "**a**", where $-1 < a < 1$, makes the graph *wider*.

Example:

$y = -2x^2$

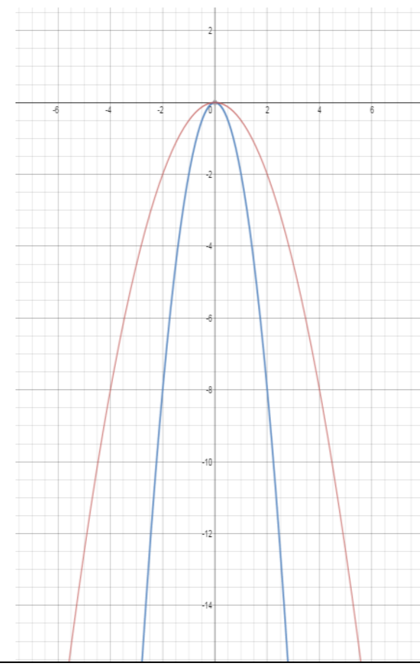
a=-2 → |a|=|-2|=2

$y = -\frac{1}{2}x^2$

a= -½ → |a|=|-½ |=½

2 > ½

$y = -2x^2$ appears as a thinner parabola because the absolute value of "a" here is greater than 1,while "**a**" in $y = -\frac{1}{2}x^2$ is a fraction.



"**b**": won't be discussed at length in this handout. Know that *increasing* **b** will shift the parabola to the *right*; and *decreasing* **b** will shift the parabola to the *left*. It affects the location of the vertex, but that is the most you need to know about **b** as far as this handout is concerned.

"**c**" **(y- intercept)**: the point where the parabola crosses the y-axis and is y's value when x = 0. Much like what we did when looking at the y-intercept in linear functions, **b**, think of the general form of the equation when x=0 :

$$f(x) = ax^2 + bx + c \rightarrow f(0) = a(0)^2 + b(0) + c \rightarrow f(0) = 0 + 0 + c \rightarrow f(0) = c \rightarrow y = c$$

<u>Example:</u>

$$f(x) = 3x^2 - 4x - 1$$

$$f(0) = 3(0)^2 - 4(0) - 1$$

$$f(0) = 0 - 0 - 1$$

$$f(0) = -1$$

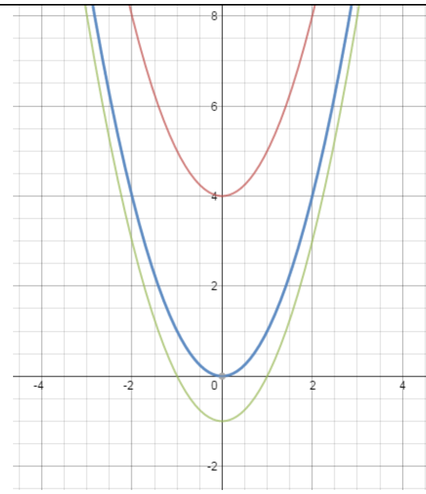
"**c**" also shifts the parabola up or down:

Let's start with $y = x^2$, the base equation, where $c = 0$

If **c** *is positive*, the parabola will shift *upwards*
If **c** *is negative*, the parabola will shift *downwards*

$y = x^2 + 4$ demonstrates the equation shifting *upwards* from $y = x^2$ (the entire parabola is shifted up by 4, which is the value of c)

And $y = x^2$-1 demonstrates the equation shifting *downwards* $y = x^2$ (the entire parabola is shifted down by 1, which is the value of c)



[Key Points]

x-Intercept – Point(s) where the graph crosses x-axis and is x's value when y=0 (which is the same thing as saying "when f(x)=0")

$$f(x) = 0 \rightarrow ax^2 + bx + c = 0$$

The x value(s) at x-intercept(s) can be found with the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You would use this when solving $ax^2 + bx + c = 0$ isn't immediate or practical.

Example:

$y = 3x^2 - 2x - 4$ Remember: a = 3, b=-2, c=-4

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-4)}}{2(-2)}$$

You will often get two points from using the quadratic formula, which makes sense when looking at the graph of a parabola. What would a graph of a parabola have to look like for there to be no x-intercepts?

Vertex – The lowest point (for parabolas that open upwards) or the highest point (for parabolas that open downwards). At the vertex, the x-coordinate is $\frac{-b}{2a}$ and y-coordinate is $f(\frac{-b}{2a})$. It is often referred to as the turning point of the parabola and also dictates where the axis of symmetry is.

Axis of Symmetry – An imaginary line that we can pretend is going vertically (up and down) through the vertex. The purpose of this axis is to see that the two sides of the parabola mirror each other and are perfectly symmetrical

<u>Note:</u> Sometimes a quadratic equation will not be written in the standard form $f(x) = ax^2 + bx + c$ that we have gone over. One form that it may appear in is called the **Vertex Form**, which is used to gain immediate information about where the point of the vertex is located. It appears in the form:

Vertex Form: $f(x) = a(x-h)^2 + k$

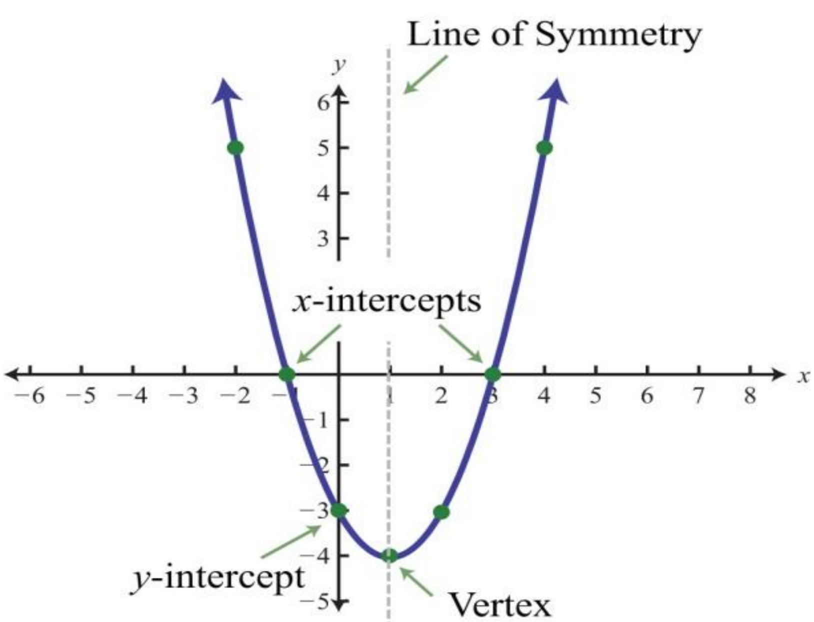
The point of the **vertex** will be located at **(h,k)**

<u>Example:</u>

$f(x) = 3(x-4)^2 + 10$

The vertex is located at (4,10)

Here is a graphical representation of all the important points we addressed.



Application:

This handout is specifically designed for Math 110, Business Calculus, and in this class a main application of linear functions is for Cost-Revenue-Profit functions. (If you need specific help with these types of functions, reference CARP's handout titled **Cost-Revenue-Profit Functions (Using Linear Equations)** as we will only be looking at these functions very briefly)

Cost functions tell us what the total cost of producing output is. The total cost consists of two different types of cost: **Variable costs** and **Fixed costs**.

Variable cost varies with output (the number of units produced).

Fixed cost will be the same no matter what the output is.

Output is defined as what/how-much is being produced and is usually represented either as the variable *"x"* or *"q"* (for quantity).

A cost function represents the **Total Cost** $(\boldsymbol{fixed\ cost + variable\ cost})$, and is often expressed as a linear equation [i.e. $y = mx + b$].

Total Cost, $\boldsymbol{C(x) \rightarrow C(x) = (m * x) + b = (Variable\ Cost * Output) + Fixed\ Cost}$

Fixed cost is assigned to b

Variable cost is assigned to m

Output is assigned to x

Example:

A factory that produces t-shirts wants to know the cost of producing 50 t-shirts in one day. They have recently calculated their costs of operation: each shirt costs $10 to make and the cost of operating equipment for the entire day is $80.
Create a cost function:

$$\boldsymbol{C(x) = (m * x) + b = (Variable\ Cost * Output) + Fixed\ Cost}$$

$$= (\$\mathbf{10} * \mathbf{50}) + \mathbf{80} = \mathbf{500} + \mathbf{80} = \$\mathbf{580}$$

It costs the company **$580** to produce 50 shirts in one day.

Works Cited

"Desmos Graphing Calculator." *Desmos*. Web. 11 Feb. 2015.

Stecker, Emily. "Features of a Parabola." *Essaying the Essay*. N.p., Feb. 2014. Web. 12 Mar. 2015.

Waner, Stefan. Costenoble, Steven. *Math for Business Analysis* (2nd Edition). Ohio: Cengage Learning (2009).