

Campus Academic Resource Program
Trigonometric Substitution Quiz

Trigonometric Substitution Quiz

Q1 $\int_0^1 \sqrt{1-x^2} dx$

Solution: Let $x = \sin(u)$

$\Rightarrow u = \arcsin(x)$

$dx = \cos(u) du$

$= \int_a^b \sqrt{1-\sin^2 u} \cdot \cos(u) du$

$= \int_a^b \sqrt{\cos^2 u} \cdot \cos(u) du$

$= \int_a^b \cos(u) \cdot \cos(u) du$

$= \int_a^b \cos^2(u) du$

$= \int_a^b \frac{1+\cos(2u)}{2} du$

$= \int_a^b \left(\frac{1}{2} + \frac{\cos(2u)}{2} \right) du$

$= \frac{1}{2} u + \frac{\sin(2u)}{4} \Big|_a^b$

$= \frac{1}{2} u + \frac{2 \sin(u) \cos(u)}{4} \Big|_a^b$

$= \frac{1}{2} \cdot \arcsin(x) + \frac{x \cdot \sqrt{1-x^2}}{2} \Big|_0^1$

$= \left[\frac{1}{2} \cdot \frac{\pi}{2} + \frac{1 \cdot \sqrt{1-(1)^2}}{2} \right] - \left[\frac{1}{2} \cdot 0 + \frac{0 \cdot \sqrt{1-0^2}}{2} \right]$

$= \pi/4$



Campus Academic Resource Program

CCRRP