

Calculus III Quiz Solutions

Q1 Find the curl of $\vec{F} = (x^2 + y^2)\hat{i} + 4y\hat{j} + z^2\hat{k}$

Solution: $\text{Curl } F = \nabla \times F$

$$\begin{aligned} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2 + y^2) & 4y & z^2 \end{vmatrix} \\ &= \left(\frac{\partial}{\partial y} z^2 - \frac{\partial}{\partial z} 4y \right) \hat{i} - \left(\frac{\partial}{\partial x} z^2 - \frac{\partial}{\partial z} (x^2 + y^2) \right) \hat{j} + \left(\frac{\partial}{\partial x} 4y - \frac{\partial}{\partial y} (x^2 + y^2) \right) \hat{k} \\ &= (0 - 0) \hat{i} - (0 - 0) \hat{j} + (0 - 2y) \hat{k} \\ \text{Curl } F &= -2y \hat{k} \end{aligned}$$

Q2 Find the area of the region R bounded by $y = x^3$ and $y = x$.

Solution: $A = \int_0^1 \int_{x^3}^x dy dx$

$$\begin{aligned} &= \int_0^1 [y]_{x^3}^x dx \\ &= \int_0^1 (x - x^3) dx \\ &= \left. \frac{x^2}{2} - \frac{x^4}{4} \right|_0^1 \\ &= \frac{1}{2} - \frac{1}{4} \\ &= \frac{1}{4} \end{aligned}$$