

Campus Academic Resource Program
Binomial Distribution

This handout will:

- Define the binomial distribution
- Discuss the Bernoulli experiment
- Outline the requirements for a binomial setting
- Offer strategies to recognize a binomial setting
- Discuss the binomial coefficient and its usage
- Discuss the binomial probability formula
- Show how to use a calculator to find the binomial probability
- Provide sample problems for practice

The Binomial Distribution

This section will:

- Define the binomial distribution
- Discuss the Bernoulli experiment

Binomial distribution is the probability distribution of the possible number of **successful outcomes** in a given **number of trials** in each of which, there is the same **probability of success**. There are specific requirements that must be met in order to have a binomial distribution, but in order to understand these requirements, we first need to understand the idea of **Bernoulli experiments**.

A random experiment that has exactly two possible outcomes is called a **Bernoulli experiment**. These outcomes are arbitrarily referred to as a “success” or a “failure”. These are arbitrary terms because a success is merely an event of interest. Below are examples of Bernoulli experiments.

Bernoulli Experiment	Possible Outcomes	
Heads from a coin flip	Success = getting a heads	Failure = getting a tails
Drawing from a deck	Success = drawing a king	Failure = not drawing a king
Purchasing a meal	Success = use a credit card	Failure = do not use a credit card

Notice that in the first example there is exactly two possible outcomes – getting a heads or getting a tails. However, in the other two examples, there are a number of different outcomes. For example, a deck of cards contain 13 different numbers/letters and 4 different suits. A person can also purchase a meal by paying in coins, bills, debit card, or credit card. But according to the Bernoulli experiment, they can both be considered to have only two possible outcomes that are complementary¹. This highlights that the term “success” and “failure” are arbitrary and is dependent upon the interest of the experiment.

Requirements for the Binomial Distribution

This section will:

- Outline the requirements for the binomial distribution
- Provide examples for the different requirements of the binomial distribution

¹ Two outcomes are considered complementary when the sum of their probabilities equal to 1 or 100%. For example, the probability of getting a “1” on a die roll is 1/6 and the probability of **not** getting a “1” is 5/6. The sum of the two probabilities equal to 6/6 or 1.

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In order to have a binomial distribution, it is necessary to meet the following requirements:

- For any individual trial, there is only **two possible outcomes** that are arbitrarily referred to as “success” or “failure”.

Example: A coin toss only has two possible outcomes: heads or tails.

- Each trial must be **independent**, meaning that the probability of one event does not influence the probability of another event.

Example: When we toss a coin two times, the probability of the first outcome does not affect the probability of the next outcome. Therefore, they are independent.

- There should be a given **number of trials** denoted as n .

Example: We decide to toss a coin 10 times to see how many times heads or tails will come up. The number (n) of trials is 10.

- The **probability of success**, denoted p , is the same in every trial.

Example: You flip a coin 10 times to determine how many times it landed on heads. The expected probability (p) of getting heads is .5 or 50% and is constant on each trial.

Recognizing a binomial setting

This section will:

- Provide strategies to recognize a binomial experiment

Recognizing a binomial distribution can be difficult at times. One of the easiest ways to identify a binomial distribution is to think about the word “**BINS**” where:

- **B** refers to **binary** meaning two possible outcomes
- **I** refers to **independent** meaning that the probabilities of each event are independent of each other
- **N** refers to the fixed **number of trials**
- **S** refers to the given probability of **success**

In the previous sections, we have talked about the definition of the binomial distribution, given the different parameters of it and finally provide tip to easily identify it. In the upcoming sections of the handout, we will focus on the different formulas of the binomial distribution by explaining them and applying them to examples.

The Binomial Coefficient Formula

This section will:

- Discuss the binomial coefficient formula and its usage

The binomial coefficient is the number of ways of picking unordered outcomes from possibilities. It allows us to know in how many ways we can arrange a number of successes (k) among a number (n) of trials. The formula of the binomial coefficient is:

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$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- $\binom{n}{k}$ is read as “n chooses k”
- $n!$ is read “n factorial”
- $n! = n \cdot (n - 1) \cdot (n - 2) \cdot (n - 3) \cdot \dots \cdot 2 \cdot 1$
- k is the number of success

Example: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Note: $0!$ is always equal to 1 and any number n that chooses 0 $\binom{n}{0}$ is equal to 1.

The following example will help us understand the binomial coefficient and apply the formula.

Example: Researchers from Harvard, Oxford, Fresh Pond Research Institute and Genzyme Genetic found that the probability for a couple to have a boy is 50%. If a couple decides to have 5 children, how many possible ways can they have those 5 children?

Quick Review Questions (answers are on the bottom of the page)

1. How many possible outcomes are there?
2. In the above example, what will represent a success?
3. What is the probability of success?
4. Finally, are the events independent?

There are many ways in which we can arrange exactly 2 boys out of 5 children depending on the order. Below are some ways to arrange exactly 2 boys out of 5 children.

- **B-B-G-G-G** meaning the two children are boys and the three last girls.
- **B-G-B-G-G** meaning the first and the third child are boys and the rest are girls.
- **B-G-G-B-G** meaning the first and the fourth are boys and the rest are girls.

Listing the number of ways we can arrange having 2 boys out of 5 children would be timely. Instead, we can use the binomial coefficient formula to solve the total number of ways that this can be arranged. Let us therefore apply the binomial coefficient formula to our example:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

n is the number of trials

k is the number of success

1) Two outcomes: boy or girl 2) The event of getting a boy 3) 50% 4) Yes

$$\begin{aligned} \binom{5}{2} &= \frac{5!}{2!(5-2)!} \\ &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1) \cdot (3 \cdot 2 \cdot 1)} \\ &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1) \cdot (3 \cdot 2 \cdot 1)} \\ &= \frac{5 \cdot 4}{2 \cdot 1} = \frac{20}{2} = 10 \end{aligned}$$

Using the binomial coefficient formula, we can see that there are 10 possible ways to arrange having exactly 2 boys out of 5 children. In order to figure this out by hand, a tree-diagram² can be used and the 10 arrangements will be as follows:

B-B-G-G-G	G-B-G-B-G
B-G-B-G-G	G-B-G-G-B
B-G-G-B-G	G-G-B-B-G
B-G-G-G-B	G-G-B-G-B
G-B-B-G-G	G-G-G-B-B

The Binomial Probability Formula

This section will:

- Discuss the binomial probability formula
- Provide example for the usage of the binomial probability formula

Binomial probability refers to the **probability** that a **binomial** experiment results in exactly X successes after n trials. In order to use the binomial probability formula, the parameters of the binomial distribution must be met. The binomial probability formula is:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

n represents the number of trials

k is to the number of successes

p refers to the probability of success in an individual trial

$(1 - p)$ is the probability of failure

$n - k$ is the number of failures

To have a better understanding of the formula and its usage, an example of a binomial setting is provided below.

² See Appendix for the tree-diagram.

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Example: A balanced die that contains six sides is tossed 10 times. What is the probability of rolling the side with the number 1 exactly 3 times?

The first question that must be answered is whether or not this meets the requirements for a binomial probability distribution.

- B – is this binary? Yes. Success is rolling a 1 and failure is not rolling a 1.
- I – is this independent? Yes. The probability of one event does not affect the probability of another event.
- N – is the number of trials given? Yes. The die will be tossed 10 times.
- S – is the probability of success given? Yes. Since there are 6 sides, the probability of rolling a 1 is $\frac{1}{6}$

Since this questions meets the requirements for a binomial probability distribution, the binomial probability formula can be used in order to find the probability of rolling a 1 exactly 3 times in 10 trials.

$$\begin{aligned}P(X = k) &= \binom{n}{k} p^k (1 - p)^{n-k} \\n = 10, k = 3, p &= \frac{1}{6} \\P(X = 3) &= \binom{10}{3} \left(\frac{1}{6}\right)^3 \left(1 - \frac{1}{6}\right)^{10-3} \\&= \left(\frac{10!}{3!(10-3)!}\right) \left(\frac{1}{216}\right) \left(\frac{5}{6}\right)^7 \\&= \left(\frac{10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1}{(3 * 2 * 1)(7 * 6 * 5 * 4 * 3 * 2 * 1)}\right) \left(\frac{1}{216}\right) \left(\frac{78125}{279936}\right) \\&= \left(\frac{720}{6}\right) \left(\frac{1}{216}\right) \left(\frac{78125}{279936}\right) \\&= (120)(.0046)(.2791) \\&= .154 \text{ or } 15.4\%\end{aligned}$$

After applying the binomial probability formula to our example, we can see that there is a .154 probability or 15.4% chance that a 1 will appear if we toss the die 10 times. Note that the computations for binomial probability formula does not need to be done by hand and rarely are they done by hand. A calculator can be used in order to calculate the binomial probability, and this handout will discuss the process in the section below.

How to Use a TI-83 for Binomial Probability Distributions

This section will:

- Show how to use the TI-83 calculator in order to solve for factorials
- Show how to use the TI-83 calculator in order to solve for “n chooses k”
- Show how to use the TI-83 calculator in order to solve for binomial probability

Solving for factorials: First, enter the number of which you are trying to find the factorial for. Then, press the MATH button (see figure 1). Four tabs will appear on the screen and they are as follows: MATH, NUM, CPX and PRB. Scroll to the **PRB** tab using the left arrow key and scroll down to “4: !” (see figure 2). Press enter and the factorial sign (!) will appear on the screen next to the number you chose. Finally, press Enter again to receive the value.

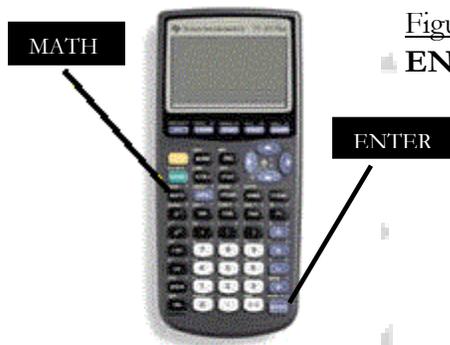


Figure 1: Shows the location of the **MATH** key and the **ENTER** key on the calculator.

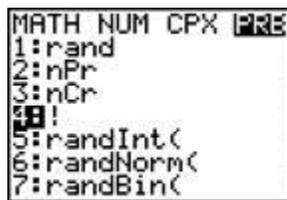


Figure 2: Shows the **PRB** tab and the factorial option.

How to input “n chooses k” $\binom{n}{k}$ ³ First, type the number the number that is represented by n . Then, press the **MATH** button. Once again, four tabs will appear on your screen (MATH, NUM, CPX and PRB). Scroll over to the **PRB** tab and pick the option “3: nCr”. The function nCr will appear on the screen to the right of the number. Lastly, type the number that is represented by k and press enter to receive the answer.



Figure 3: Shows the **PRB** tab and the nCr option.

³ $\binom{n}{k}$ “n chooses k” can be represented as nCr or nCk

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Figure 5: displays the screen after you enter the number represented by k .

Binomial probability using the calculator: First, press the 2^{nd} button then press the **VAR** key. Two tabs (**DISTR** and **DRAW**) should appear on your screen. Stay on the **DISTR** tab and scroll all the way down to “**8: binompdf**” and press **ENTER**.

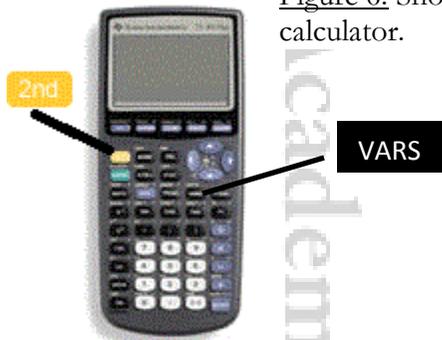


Figure 6: Shows the location of the 2^{nd} key and **VAR** key on your calculator.

After you press the **ENTER** key, the function “**binompdf**” should appear on your screen. Next, you will need to enter the numbers that represents the number of trials n , the probability of success p , and the number of success X respectively.

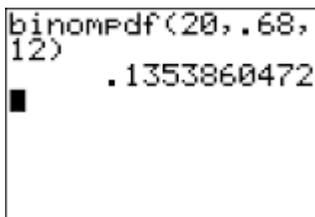


Figure 8: Shows the screen on your calculator after you pressed the **binompdf**(option and entered the numbers corresponding to n , p and X . In this case, $n = 20$, $p = 0.68$, $X = 12$.

Sample Problems

Suppose you have an 87% chance of making a free throw in basketball. If you attempt to shoot the basketball exactly 12 times, what is the probability that you will successfully make exactly 9 of them?

We will solve the first problem then let you do the rest of the examples on your own.

- First, we want to know if this experiment follows the Bernoulli principle and if we have a binomial distribution. The random experiment of shooting a free throw means **you either make it or you do not make it**. This follows the Bernoulli principle, therefore we can affirm that yes we have a binomial distribution.
- The second thing we want to know is whether the events are independent meaning the probability of successfully making one free throw does not change the probability of successfully making the next one. In this case, we can **say the events are independent** because if the first shot is a successful free throw, it does not affect the probability that the next shot will be made successfully or unsuccessfully.
- Third and fourth step are to know if we have a probability of success and a number of trials. Looking at the problem, both the probability of success (.87) and the number of trials (12 free-throw attempts) are stated in the problem.

Now that all the parameters are met, we can go ahead and perform the calculation using the binomial probability formula.

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$P(X = 9) = \binom{12}{9} \times (.87)^9 \times (.13)^3$$

$$P(X = 9) = 220 \times (.2855) \times (.0022)$$

$$P(X = 9) = .138 \text{ or } 13.8\%$$

After using the binomial probability formula, we know that in 12 attempts there is a 13.8% chance of successfully making exactly 9 of them.

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Sample 1: Consider a large population where 32% are registered to vote. In a Simple random sample of size 5, what is the probability that exactly 3 are registered voters?



Sample 2: Suppose you have 8 trials, in how many ways can you arrange to have 3 successes and 5 failures?

Sample 3: A student takes a multiple choice test that only has exactly 5 questions. The student failed to study for the exam, so he decides to randomly guess on each of the questions. There are 5 choices for each problem (A, B, C, D, and E) so this means that there is a 20% chance of getting a question correct. Let X equal the number that the student answers correctly. What is the probability that $X = 2$?

Sample 4: (Refer to the previous problem). Suppose you were given a test that has 10 questions. Again, each question has 5 choices (A, B, C, D, E). What is the probability of passing this test after randomly guessing on each item on the exam? (Note: A passing score for this test is a 70% or higher).

Sample 5: A company is looking to hire two new associate program coordinators. After a couple of weeks, the company receives 20 applications. Of the 20 applications, 13 are women and 7 are men. What is the probability that the company selects both women for the position that needs to be filled?

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Sample 1 15.2% **Sample 2** 56 **Sample 3** 20.5% **Sample 4** 0.08% **Sample 5** The problem does not meet the parameters for the binomial distribution. The trials are not independent. Probability of selecting a woman for the first slot is 65% ($13/20$), and the probability of selecting a woman for the second slot is 63.2% ($12/19$)

Answers:



COARRP

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Works Cited

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