Handout Objectives:

This handout will:

- Introduce the unit circle.
- Define and describe the unit of radians.
- Show how to convert between radians to degrees.

The Unit Circle:

This section will define the unit circle

Trigonometry is based upon a geometric object called the **unit circle**, a circle with a radius of 1, centered at the origin. We can plot the **unit circle** using the equation $x^2 + y^2 = 1$, as shown in figure 1 below:

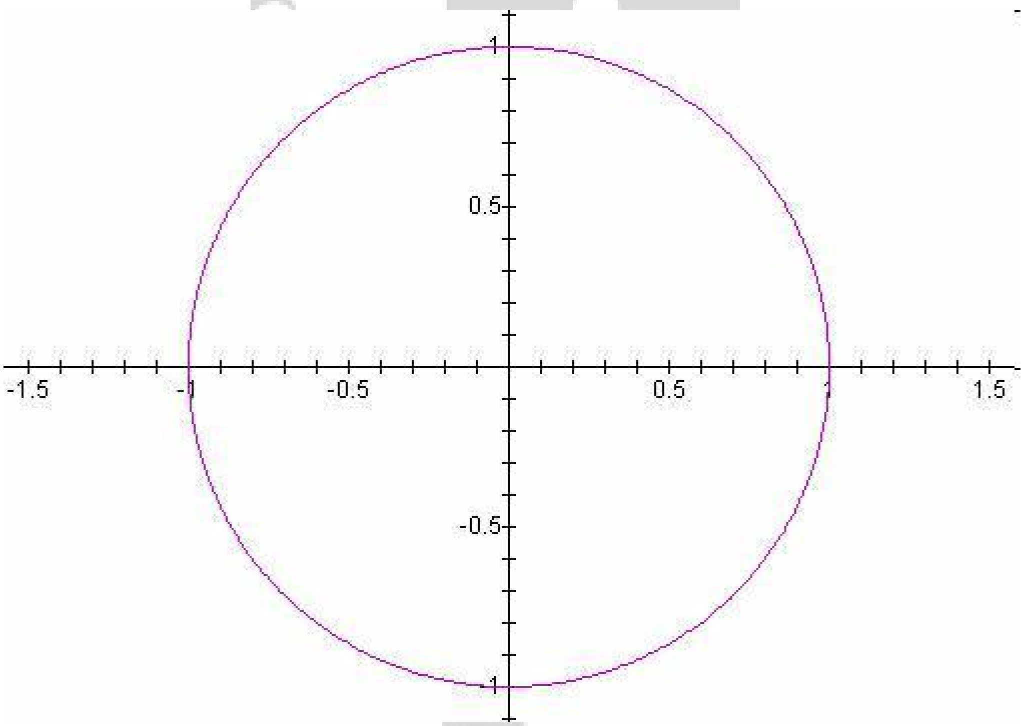

Figure 1: *The figure shows the unit circle. The unit circle has a radius of 1 and its center is at the origin.*

We measure an angle on the unit circle with respect to the positive portion of the x-axis, where our angle is defined to be zero. That is, to measure an angle, we sweep out a certain distance along the arc of the circle, and measure the corresponding angle from the x-axis. See figure 2 on next page.

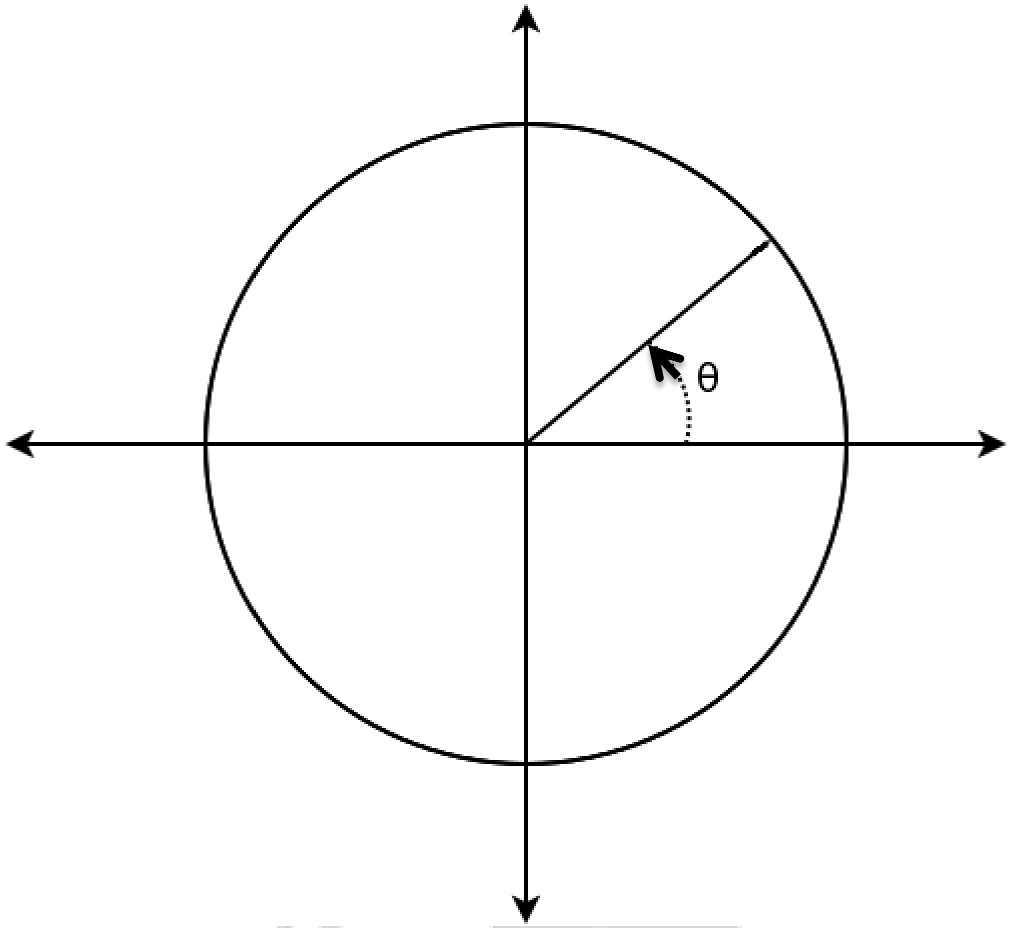

Figure 2: *The figure shows how the angles are measured in a unit circle. The angle is measured relative to the positive x-axis.*

This angle is often denoted by the variable θ (theta) and is measured in units of degrees or radians. You may be more familiar with using degrees, for they are the standard unit of measurement in geometry (where one degree is an angle extending $\frac{1}{360}$ around the circle). Radians may be more useful in certain mathematical applications. For example, the formula for calculating the area of a slice of a circle, $A = \frac{r^2}{2}\theta$, can only be written like so when angles are in radians.

Radians:

This section will…

- Define radians graphically and algebraically
- Introduce formulae to convert from radians to degrees and vice-versa.

To understand the meaning of a radian, imagine a string as long as the radius of a circle. For the unit circle, this string would have a length of 1. If we position this string along the circumference of the circle, it would sweep out a particular angle. The angle formed by undergoing this process is one radian. This can be seen as follows in Figure 3[1]:

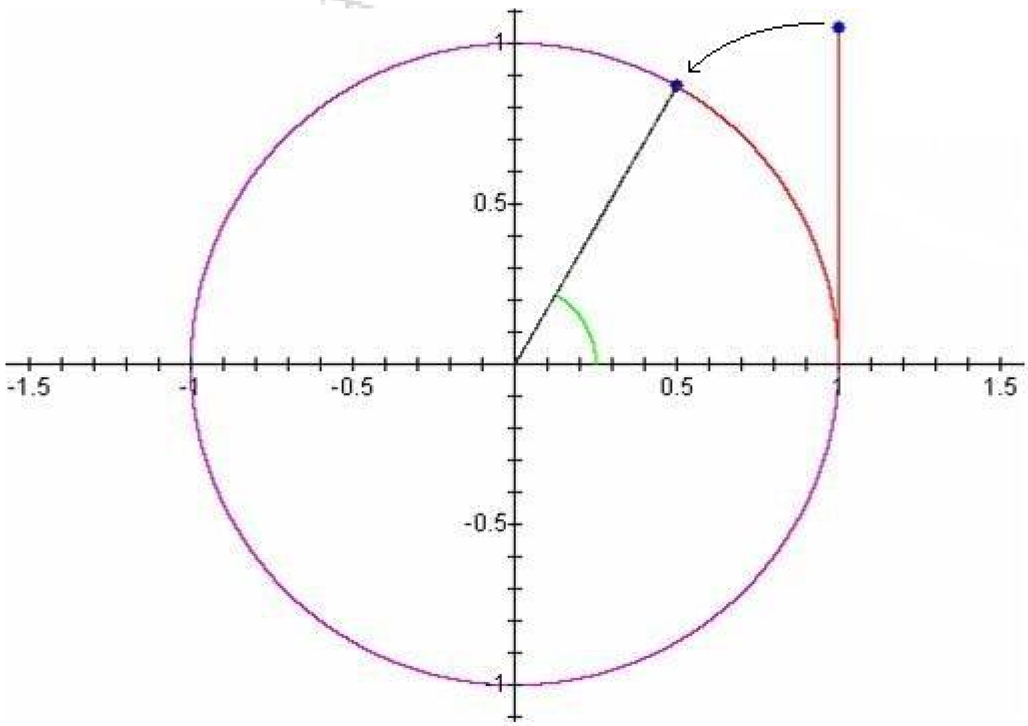

Figure 3: *The figure illustrates an angle of one radian. The angle is depicted by the green curve. Note that the arc length and radius are equal when the angle is one radian.*

- In other words, the arc length of the string equals the circle's radius opposite to an angle of one radian.

If this diagram doesn't make sense to you, another option is to go to this link, http://upload.wikimedia.org/wikipedia/commons/4/4e/Circle_radians.gif , which visually explains how radians work. It may load in the middle, but let it finish and restart; it's very short.

[1] Image may not be clear if printed in black-and-white.

More generally, radians may be defined as the ratio of the *arc length* to the *radius*. If S denotes arc length, then the angle in radians is

$$\theta \equiv \frac{S}{r}$$ **Definition of Radians**

For example, if we measure the arc length around the entire circle, then the arc length is equal to the circumference. The circumference of a circle is $2\pi r$. In this case the angle of one rotation about a circle is

$$\theta_{\text{full circle}} = \frac{2\pi r}{r}$$
$$= 2\pi$$

If we measure the arc length around a quarter of the circle, then we measuring a quarter of the circumference or $S = \frac{2\pi r}{4} = \frac{\pi}{4}r$, so

$$\theta_{\text{quarter circle}} = \frac{2\pi r/4}{r}$$
$$= \frac{2\pi r}{4r}$$
$$= \frac{\pi}{2}$$

In both of these examples, the r drops out. In general, radians are independent of the radius of the circle. As such, we typically measure radians using the unit circle, where $r = 1$. In this context, radians can be thought as 2π times the fraction of the circle you are measuring. So if you transfer half a circle, then

$$\theta_{\text{half circle}} = \frac{1}{2} \cdot 2\pi$$
$$= \pi$$

Where the $\frac{1}{2}$ comes from the fact we are measuring the arc length of *half* the unit circle.

A graph of the unit circle is shown on the next page, containing numerous angles in degrees and their corresponding values in radians.

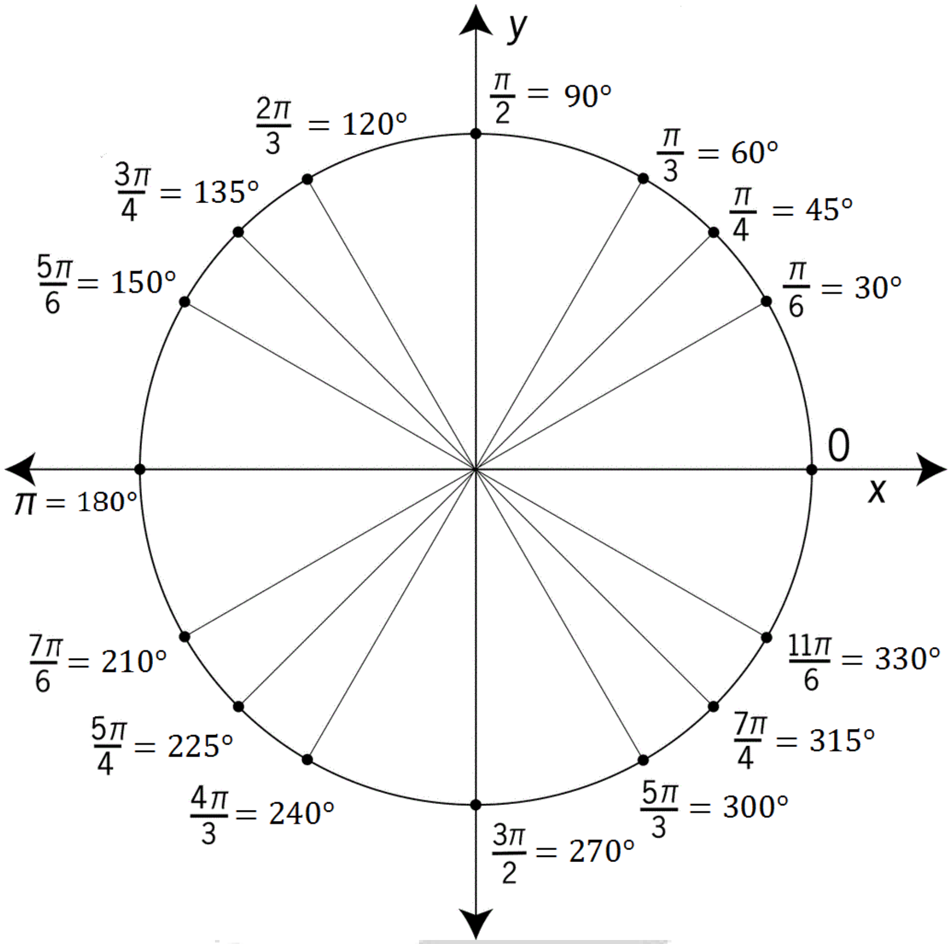

Figure 3: *The figure shows the angles (both in radians and degrees) in different rotations in a circle. Note that these rotations begin at the x-axis, where the angle is zero.*

Although this graph illustrates some equivalent values in degrees and radians, we can determine these values without referring to the graph. If we want to convert from degrees to radians, we may do so using this formula:

$$\theta_{\text{Radians}} = \theta_{\text{Degrees}} \times \frac{\pi}{180°}$$

In a similar fashion, we can convert radians into degrees using the following formula:

$$\theta_{\text{Degrees}} = \theta_{\text{Radians}} \times \frac{180°}{\pi}$$

Examples:

Convert the following angles from degrees to radians, or radians to degrees.

a. $65°$ to radians.

1) Converting an angle from **degrees** to **radians**, requires us to multiply the degrees value by $\frac{\pi}{180°}$.

$$\theta_{\text{Radians}} = 65° \times \frac{\pi}{180°}$$

2) In this equation, we divide $65°$ by $180°$

$$\theta_{\text{Radians}} = \pi \times \frac{65°}{180°}$$

3) Then, we simplify the fraction $\frac{65°}{180°}$.

$$\theta_{\text{Radians}} = \pi \times \frac{13}{36}$$

$$\theta_{\text{Radians}} = \frac{13\pi}{36}$$

b. $\frac{5\pi}{6}$ radians to degrees

1) Converting any value in **radians** to **degrees** requires us to multiply the radian value by $\frac{180°}{\pi}$.

$$\theta_{\text{Degrees}} = \theta_{\text{Radians}} \times \frac{180°}{\pi}$$

$$\theta_{\text{Degrees}} = \frac{5\pi}{6} \times \frac{180°}{\pi}$$

2) The π in the numerator and in the denominator cancels. Multiplying $\frac{5}{6}$ and $180°$ gives us our answer in degrees.

$$\theta_{\text{Degrees}} = \frac{5}{6} \times 180°$$

$$\theta_{\text{Degrees}} = \frac{900°}{6}$$

$$\theta_{\text{Degrees}} = 150°$$

Works Cited:

Bodrero, J. (n.d.). Unit Circle Trigonometry. Retrieved March 6, 2015, from http://www.snow.edu/jonathanb/Courses/Math1060/unit_circ_trig.html

File:Circle radians.gif. (n.d.). In Wikipedia. Retrieved December 9, 2009, from http://en.wikipedia.org/wiki/File:Circle_radians.gif