Campus Academic Resource Program

Introduction to Trigonometry

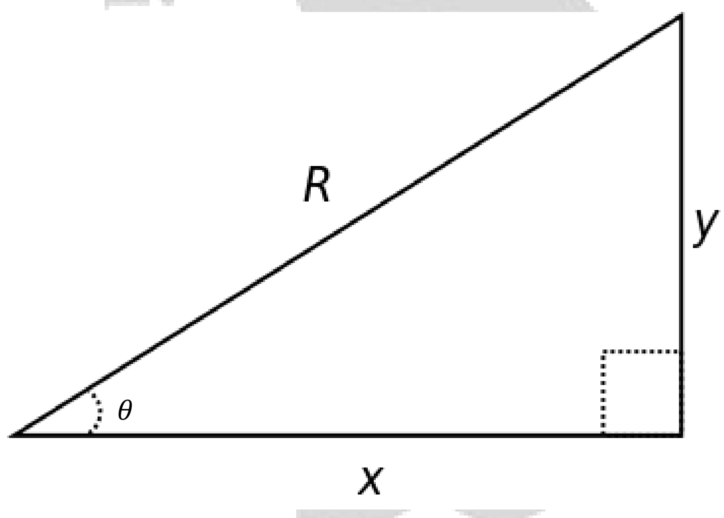
This handout will:

- Provide triangular definitions for trigonometric functions.
- Show how to use the trigonometric functions to find properties of right triangles.

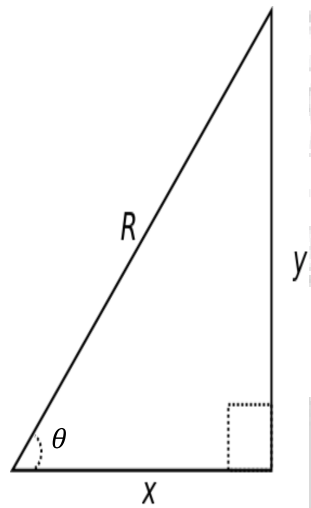
The development of trigonometry began as an extension of geometry, created for the purpose of astronomical observation. However, it has since become a branch of mathematics that has applications in many different fields, making it an important subject of study. This handout will begin by defining sine, cosine, and tangent in the context of relating lengths and hypotenuses of right triangles.

Definitions of Sine, Cosine, and Tangent:

This section will define sine, cosine, and tangent. Although more trigonometric functions exist in a typical pre-calculus class, the three functions introduced in this handout are sufficient for a first look at trigonometry. To define these functions, we examine a right triangle:



The triangle has sides of length x, y, and hypotenuse R. The length of the sides x and y are determined by the *angle* we used when constructing the right triangle. To see this, imagine what happens when we consider a triangle with a larger angle.



As we can see, the larger angle has resulted in a smaller value of x, and a larger value of y. This suggests that there is a relationship between the angle in the triangle and the corresponding lengths of the triangle's sides. To quantify the relationship between angles and lengths, we define three particular functions called sine, cosine, and tangent, which are the ratio of the triangle's sides. If we let x be the adjacent side on the triangle, y be the opposite side on the triangle, and R be the hypotenuse, then:

$$\text{sine} = \frac{\text{Opposite}}{\text{Hypotenuse}} \longrightarrow \sin(\theta) = \frac{y}{R}$$

$$\text{cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \longrightarrow \cos(\theta) = \frac{x}{R}$$

$$\text{tangent} = \frac{\text{Opposite}}{\text{Adjacent}} \longrightarrow \tan(\theta) = \frac{y}{x}$$

On the left is the written description of what the three functions are, and on the right is the mathematical form they are expressed in. The symbol θ (theta) indicates the angle formed on the unit circle, and resides within the parentheses. Be aware that the parentheses in this case do not indicate multiplication, but instead refer to the *input* of the function.

We can solve the above equations for x and y to find another useful formulation:

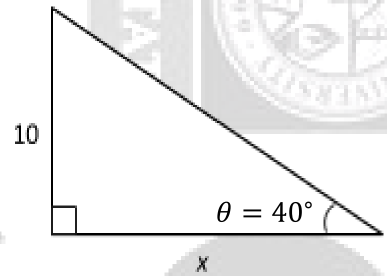
$$x = R\cos(\theta)$$

$$y = R\sin(\theta)$$

Important: Sine, cosine, and tangent are defined *only* for right triangles; that is, you cannot apply these definitions of sine, cosine, and tangent to an obtuse or acute triangle.

Examples:

a) Find the length x for the right triangle shown.



To solve for x, we first recognize that this triangle is a right triangle—this enables us to use sine, cosine, and tangent. The next step is to identify which trigonometric function we will use, which we infer by considering what pieces of information are given to us. We see that we are given values for the angle, as well as the side opposite to the angle. Also, the value we are looking for is the length adjacent to the angle. This suggests that we will want to choose a trigonometric function that relates both the opposite and adjacent sides. By referencing the equations on the previous page, we can see that tangent relates an angle to the opposite and adjacent side:.

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan(40°) = \frac{10}{x}$$

Solving this for x gives:

$$x = \frac{10}{\tan(40^0)}$$

Using a calculator, we find $\tan(40^0) \approx 0.84$

$$x \approx \frac{10}{0.84}$$

$$x \approx 11.9$$

So the length of the adjacent side is 11.9.

Practice Problems:

a) Find the value of R for the following triangle: