Scientific Notation and Exponentiation

This handout will:

- Explain significant figures
- Explain exponentiation and some of its properties.
- Explain scientific notations.
- Provide examples that show techniques used in solving problems with exponentiation.

Significant Figures

Significant figures are digits in a number that express the precision of the number. The following are significant figures in a number:

- Nonzero digits.
- Zeros between significant digits.
- Trailing zeros if the number has a decimal point.

Digits that are not significant exist only to show the scale of a number. These digits serve as place holders and do not contribute to the number's accuracy. The following are not significant figures:

- Leading zeros. These are zeros that are to the left of all nonzero digits.
- Trailing zeros if the number does not have a decimal point.

In the presence of a decimal point, any zeros trailing all nonzero digits will make the whole number precise up to the last zero; these digits contribute to the number's accuracy. Without a decimal point however, zeros trailing the nonzero digits exist only to show the scale of the number, and are therefore insignificant. Also, leading zeros only shows the position of the decimal point and do not make the number more precise.

Let's now take a look at the following set of examples.

<u>Example</u>: Determine the significant figures in the following numbers.

Number	Significant Figures
987.65	• All digits are significant since all are nonzero digits.
34000	• Only 3 and 4 are significant figures. • The trailing zeros are not significant since there's no decimal point.
340.0	• 3, 4 and the two 0s are significant. • Since there's a decimal point, all trailing zeros are significant.

00340.00	• 3, 4 and the three trailing zeros are significant. • The two leading zeros are not significant.
36000005	• 3, 6, 5, and the five zeros in between are all significant.
0.0000012	• Only 1 and 2 are significant. • The six leading zeros are not significant.

Exponentiation

Exponentiation is a mathematical operation written as x^n, where x is the base number and n is the exponent (also known as index or power). Exponentiation corresponds to repeated multiplications such as $x^3 = x \cdot x \cdot x$, where $n = 3$. Here are algebraic properties of exponentiations:

Exponent Properties	Explanations	Examples
$x^m x^n = x^{m+n}$	• Multiplying x to the power m by the x to the power n gives us x with the exponent $(m+n)$ • m and n can be any number.	$(5)^2(5)^3 = (5)^{2+3} = 5^5$
$(x^m)^n = x^{mn}$	• When the entire expression x to the mth power is raised to the nth power, we get x to the power of $m \times n$.	$(3^2)^4 = 3^{2\times4} = 3^8$
$x^m y^m = (xy)^m$	• $x \neq y$ • x and y have the same exponents m • Multiply x and y. The product will be raised to the mth power.	$(3)^3(2)^3 = (3 \times 2)^3 = 6^3$
$x^0 = 1$	• Any number x to the zeroth power is just 1.	$987654^0 = 1$
$x^{-m} = \frac{1}{x^m}$	• Any number x with a negative exponent m is 1 over x to the positive mth power.	$9^{-4} = \frac{1}{9^4}$
$\frac{x^m}{x^n} = x^{m-n}$	• x to the mth power divided by the x to the nth power results in x to the $(m-n)$ power. • This only works when the base is the same in the numerator and the denominator.	$\frac{5^7}{5^4} = 5^{7-4} = 5^3$

Now that we've seen some exponentiation properties and how they're used, let's take a look at some examples demonstrating these properties.

Example: Simplify the following expression.

$$(5^3)^4 \times (5^5)^{\mathbf{2}}$$

Now, you may be wondering how to approach this problem. We can start by simplifying these two expressions.

$$(5^3)^4 = (5)^{3\times4} = 5^{12}$$

$$(5^5)^{\mathbf{2}} = (5)^{5\times2} = 5^{10}$$

We now have the following.

$$5^{12} \times 5^{10}$$

Because the base numbers are the same, we can simplify as follows to get our answer:

$$5^{12} \times 5^{10} = (5)^{12+10} = 5^{22}$$

$$\Rightarrow (5^3)^4 \times (5^5)^{\mathbf{2}} = 5^{22}$$

Scientific Notation

Scientific notation is a way to express very big or very small numbers by using exponents. A number written in scientific notation has the form,

$$x \times 10^n$$

where,

- x is any real number in decimal form satisfying $1 \leq |x| < 10$.
- n is an exponent (also known as power or index) of 10.
    - n can be viewed as the number of times the decimal point was moved to get the value of x.

<u>Example</u>: Consider the following set of multiplications showing $1{,}234$ expressed as $x \times 10^n$.

$$123.4 \times 10^1 = 1{,}234$$

$$12.34 \times 10^2 = 1{,}234$$

$$1.234 \times 10^3 = 1{,}234$$ ⇐ Note that, because of our definition of scientific notation, this is the only number in the correct form!!

$$12{,}340 \times 10^{-1} = 1{,}234$$

Moving the decimal point to the left results in a positive power of 10, while moving the decimal to the right results in a negative power of 10.

The following are examples of conversions to scientific notations.

Example: Convert the following to scientific notation.

$$39876$$

Since the decimal point isn't shown, the decimal point is to the right of value of the lowest place, 6. We move the decimal point until we get a number between 1 and 10.

$$39876.0$$

$$39876.0 \rightarrow 3.9876$$

We have $x = 3.9876$. Now, we multiply this value by 10^n. Since the decimal point was moved to the left 4 times, $n = 4$. Therefore,

$$x \times 10^n$$

$$\Rightarrow 39876 = 3.9876 \times 10^4$$

Let's take a look at another example on converting numbers to scientific notation.

Example: Convert the following number to scientific notation.

$$0.000023$$

Move the decimal point to the right 5 times.

$$0.000023 \rightarrow 2.3$$

Here, we have $x = 2.3$. We then multiply this number by 10^n. Since the decimal point was moved 5 places to the right, we have $n = -5$.

$$x \times 10^n$$

$$\Rightarrow 0.00023 = 2.3 \times 10^{-5}$$

Now that we've examined how to write numbers using scientific notation, let's take a look at how to multiply numbers written in scientific notation. In multiplying scientific notations, we must follow the properties of exponentiation and our result must also be in scientific notation. Let's briefly review this concept.

Multiplication of Scientific Notation

The following formula shows how to multiply numbers in scientific notation.

$$(a \times 10^n) \times (b \times 10^m) = (a \times b) \times 10^{m+n}$$

- Here, multiply a and b.
- The exponent of 10 is the sum of its exponents on both scientific notations.
- If in the resulting scientific notation, $a \times b$ does not satisfy $1 \leq |a \times b| < 10$, write $a \times b$ using scientific notation.

The following examples demonstrate multiplication using scientific notation.

Example: Multiply the following scientific notation.

$$(6 \times 10^3) \times (9 \times 10^5)$$

Multiplying 6 and 9, and 10^3 and 10^5, we get,

$$(6 \times 9) \times (10^3 \times 10^5) = 54 \times 10^{3+5} = 54 \times 10^8$$

Since 54 is larger than 10, write 54 using scientific notation.

$$54 = 5.4 \times 10^1$$

$$54 \times 10^8 = (5.4 \times 10^1) \times 10^8 = 5.4 \times (10^1 \times 10^8) = 5.4 \times (10^{1+8}) = 5.4 \times 10^9$$

Therefore,

$$(6 \times 10^3) \times (9 \times 10^5) = 5.4 \times 10^9$$

Citation

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